Dynamic RAT Selection and Pricing for Efficient Traffic Allocation in 5G HetNets

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Abstract-In this paper, we focus on 5G heterogeneous networks, considering the existence of multiple Distributed Units (DUs) that can provide access to end users implementing several access technologies, managed by a Central Unit (CU) responsible for the allocation of network resources. Based on a distributed dynamic pricing scheme, that gives to User Equipment (UE) the ability to select the appropriate Radio Access Technology (RAT) depending on its sensitivity to congestion, we investigate a scheme of greater granularity, where UEs are able to allocate each of their traffic classes to the appropriate RAT, exploiting their multi-homing features. As UEs are sequentially polled to request for network resources, we develop a centrally controlled proportionally fair ranking as a benchmark policy. We then propose a dynamic polling policy that presents close performance to the benchmark policy, while maintaining a distributed nature. We evaluate our framework for a variety of traffic classes in terms of Quality of Service (QoS) requirements, and we provide results on capacity utilization and load distribution over available RATs, as well as access price variations.

I. INTRODUCTION

Ultra-Dense Heterogeneous Networks (UDHetNets) are expected to support the transition to the 5th Generation (5G) of mobile network access by offering dense network setups and a plethora of different networks for the user devices to associate to [1]. Based on their application needs for network transfer, the network selection becomes a problem of critical importance; for example, safety applications should be prioritized over the network, but using networks that operate in the unlicensed frequency spectrum bands may not ensure that the message is delivered timely. Therefore, the selection of the appropriate Radio Access Technology (RAT) that should be used is of paramount importance.

The provisioning of such network connectivity over multiple RATs is assuming the existence of multi-homed clients; in fact, contemporary devices are able to associate to multiple networks at the same time (e.g. LTE, fallback mode to UMTS and WiFi concurrently), thus rendering the concurrent transfer of data over multiple interfaces feasible. As a matter of fact, several protocols have emerged that support multi-homing features. The two most notable examples are Multipath-TCP (MP-TCP) and Stream Control Transmission Protocol (SCTP). Switching to the operator's side, hooks for selecting the RAT that will serve the clients of the network are present in the recent standardization work for 5G-New Radio (5G-NR) [2]. In the detailed architecture, the base station components are

disaggregated to Central Units (CUs) and Distributed Units (DUs). The DUs may offer several heterogeneous network connections (e.g. 5G-NR, LTE, WiFi), managed through a single CU and allowing the operator to select the technology that will be used to forward the traffic to the User Equipment (UE) of the network [3].

Nevertheless, RAT selection is subject to a number of different parameters. For example, selecting an IEEE 802.11ac network over LTE may promise higher speeds for the wireless part, but this is only valid in the case of low interference and contention. Similarly, for the case of just transmitting email traffic, a legacy UMTS connection suffices for most of cases. If we bring into the picture the fact that cellular connections are metered, contrary to free WiFi hotspots that are found in abundance, the problem of selecting the appropriate RAT to transmit the client traffic becomes more complex.

In this work, we formulate the problem of distributed RAT selection by considering multiple traffic classes (TCs) over different heterogeneous technologies for serving the different applications running at the UEs of the network. In our prior work [4], we showed how UEs can distributively take such decisions, applying three different policies for ordering the UEs that define the polling order for the allocation of network resources. The pricing scheme applied is inspired from the Paris Metro Pricing (PMP), a service differentiation scheme that was first used in Paris metro to give to its passengers the ability to opt for less congested wagons. Here, we extend this solution based on a centrally controlled fairness scheme for the UEs, by ordering them based on weights indicating their sensitivity to traffic congestion (depending on the under study technology) and polling them based on the distributed policy that is closer to the centralized approach. We derive the proportionally fair ranking of the UEs and propose a dynamic policy selection scheme for polling the clients of the network.

The rest of the paper is organized as follows: Section II provides a literature overview of other approaches for the RAT selection problem. In Section III we describe our system model for deriving the technologies that will be used per each client. Section IV presents our experimental setups and results that we receive for different policy settings. Finally, in Section V we conclude our work.

II. RELATED WORK

RAT selection has been a widely investigated problem that has gained attention lately for UDHetNets. A simple solution for determining the RAT that the network's UEs shall associate to, is by assigning prices for each available technology. The rise of heterogeneous networks as a solution to the problem of capacity crunch, reinstates the pricing problem, this time with the existence of multiple traffic classes per mobile user that can be concurrently allocated to different access technologies. An extensive classification of network selection policies for HetNets was provided in [5]. Similarly, a reward based algorithm is presented in [6], where rewards received from the base stations are used by the users to independently update the traffic that is sent to each traffic class per available technology. In [7], the multi-user RAT selection problem is modeled as a non-cooperative game, where each user tries to selfishly maximize its own throughput, and the impact of a user's decisions on other users performance is investigated. The average number of per-user RAT switchings is used as a metric to evaluate the convergence time of the proposed algorithms. A model for establishing on-demand multi-RAT conditions is proposed in [8], where mobile users form short range mesh networks and collaborate, by sharing their Internet access with provision for proper routing policies with loadbalancing and fairness. A pricing-based scheme for concurrent uplink access through LTE and WiFi is proposed in [9]. In [10], the coexistence of small cell service providers with macrocell providers is considered.

An initial work for the application of Paris Metro Pricing (PMP) in the context of packet delivery networks was provided in [11] as a solution to the congestion control problem for differentiated classes of service levels. Similarly in [12] and [4], we investigated its applicability in Multi-RAT deployments, for several traffic classes per UE served by the available access technologies. These works concluded in the application of three different policies for the manner of polling the clients and calculating the prices per each technology. In this work, we extend the scheme in order to include a policy based on ordering the Multi-RAT UEs according to the Kendall tau distance from the weighted proportionally fair ranking, that is optimal, and apply a dynamic scheme for selecting RATs per each traffic class per each client.

III. RAT SELECTION IN HETEROGENEOUS NETWORKS

The problem of radio access technology selection has attracted much research effort leading to a variety of decision algorithms. The most important metric that drives the decisions of the proposed algorithms is the SINR, based on which, the outage probability and the average rate can be computed [13]. In LTE-A, access decisions are based on the Reference Signal Received Power (RSRP) and the Reference Signal Received Quality (RSRQ). RSRP measures the power of the LTE reference signals spread over the full bandwidth and narrowband, while RSRQ considers RSSI and the number of the used resource blocks measured over the same bandwidth.

In handover procedures, the LTE-A specification provides the flexibility of choosing between RSRP, RSRQ, or both.

A. Distributed RAT Selection

While measurements based on received power provide a tangible approach for efficient access technology selection, the inclusion of low power and low range access options like Small Cells does not make power-based decisions efficient. A UE under a Small Cell coverage would take a decision to associate with a higher power macrocell even if it is more congested than the Small Cell. This decision would have direct impact on the energy efficiency of the UE, in addition to the lost opportunity for more network resources. To solve this problem, algorithms that add cell selection bias have been proposed [14]. A bias, virtually expanding the range of a low power access option, is added to the UE's received power based metrics, pushing them into lower power ones.

The introduction of heterogeneity as a key element of the 5G networks architecture [15], only adds to the complexity of access decisions. The availability of several radio access technologies in an heterogeneous network landscape calls for more intelligent access models. To this end, we developed a dynamic pricing framework based on [12] and [4], enhancing the Paris Metro Pricing scheme, that was first introduced for congestion control of the metro wagons of the Paris underground transportation network with static price differentiation.

B. Dynamic Pricing Framework for Heterogeneous Networks

In the first part of the framework, we considered a heterogeneous network of multiple access technologies (LTE, UMTS, WiFi), where each UE could choose the access technology for its traffic that maximizes its utility function, expressed as:

$$U_i(m) = V - p_m - \theta_i f(Q_m, \bar{C_m}) \tag{1}$$

where V is a constant price for accessing the multi-technology service, p_m is the current price for accessing technology m, θ_i is the sensitivity to congestion of UE_i and $f(Q_m, \bar{C}_m)$ is a function for the perceived congestion at class m. Congestion perception is a function of the mass of UEs in access technology m denoted as Q_m and of its available capacity \bar{C}_m . Setting a maximum and a minimum price for each technology, for the cases of non-available and fully available capacity respectively, a dynamic price per access technology is formed, as a function of its load:

$$p_m = \max\left(p_m^{\min}, p_m^{\max}\left(1 - \frac{\bar{C}_m}{C_m}\right)\right) \tag{2}$$

where C_m is the total capacity of access technology m. Aiming to maximize their utility functions, UEs choose the appropriate access technology with the minimum price that concurrently satisfies their Quality of Service (QoS) requirements. By introducing dynamic pricing in the Paris Metro Pricing scheme, where decisions are made distributively by the UEs based on their utility functions, an autonomous access technology selection algorithm is provided for heterogeneous networks, proved to lead to Wardrop equilibrium [16].

In the second part of the framework, we extended the proposed dynamic pricing scheme to also consider multiple traffic classes per UE, with different QoS needs to represent the diverse requirements of end-user applications. In this part we associated each traffic class of UEs with a similar utility function characterized by the sensitivity of each traffic class:

$$U_i^l(m) = V - p_m - \theta_i^l f_m(Q_m, \bar{C}_m)$$
(3)

For each traffic class, a UE chooses the RAT that maximizes the corresponding utility function. In this case, $\theta_i^l = r_i^l/w_i$ represents the sensitivity of UE_i's traffic of type l, with r_i^l being the data rate demand of UE_i for its traffic class l, and w_i the normalized spectrum efficiency of UE_i, with $w_i \in (0,1]$, which is used to abstract the physical layer for the channels of the available RATs. Additionally, we provided each UE with the capability of multiple concurrent associations. Moreover, we extended the system model with multi-homed UEs and we included higher throughput 5G technologies (WiGig). We assessed this enhanced pricing scheme with a real testbed evaluation [4].

C. Centralized System Design

To assess the proposed dynamic Paris Metro Pricing based resource allocation framework, we compare it with a centralized system, where decisions are taken based on UEs traffic needs with a weighted proportionally fair approach. In this Section, we provide a detailed analysis of the centralized access decision that is based on a weighted proportionally fair solution. We formulate and solve the centralized scheme as a utility maximization problem and provide the optimal solution for weighted proportionally fair bandwidth allocation, and the UEs are then ranked in descending order of allocated bandwidth. We compare this optimal ranking to the distributed dynamic PMP allocation ranking in terms of pairwise disagreements using Kendall tau distance [17].

Every UE may have bandwidth needs for up to L traffic classes. Each traffic class presents traffic sensitivity θ_i^l , and the average traffic sensitivity of UE $_i$ is defined as $\overline{\theta}_i = \sum_{l=1}^L \theta_i^l/(\mathbf{1} \cdot \mathbf{d}_i)$, where $\mathbf{d}_i = \{d_i^1,...,d_i^L\}$ and

$$d_i^l = \begin{cases} 1 & \text{if UE}_i \text{ has traffic class of type } l \\ 0 & \text{if UE}_i \text{ has no traffic class of type } l \end{cases}$$
 (4)

Assuming that N UEs are served by the HetNet, each UE $_i$ is allocated bandwidth equal to c_i , i=(1,...,N), such that $\sum_{i=1}^N c_i \leq C$, where C is the total system capacity. We need such an allocation that is proportionally fair over the average traffic sensitivities $\overline{\theta}_i$ of the served UEs by the HetNet. Based on the definition of proportional fairness by Kelly et al. [18], a vector of rate allocation $\mathbf{c}=(c_1,...,c_N)$ is proportionally fair if it is feasible, that is $\mathbf{c}\geq 0$ and $\sum_{i=1}^N c_i \leq C$ and if for any other feasible vector \mathbf{c}^* , the aggregate of proportional changes is zero or negative and is expressed as

$$\sum_{i=1}^{N} \overline{\theta}_i \frac{c_i^* - c_i}{c_i} \le 0 \tag{5}$$

that can be also expressed as

$$\sum_{i=1}^{N} \overline{\theta}_i (\log(c_i))' dc_i \le 0 \tag{6}$$

We observe from (6) that the proportionally fair allocation solution maximizes the utility function $U_i(\mathbf{c}) = \sum_{i=1}^N \overline{\theta}_i(log(c_i))$. Thus, in order to find the proportionally fair solution we have to solve the following maximization problem

$$\max_{\mathbf{c}} \qquad \sum_{i=1}^{N} \overline{\theta}_{i} log(c_{i})$$
subject to
$$\sum_{i=1}^{N} c_{i} \leqslant C$$
and
$$c_{i} \geqslant 0, \forall i = 1, ..., N$$
 (7)

The problem in (7) has a unique solution since the utility function is strictly concave and the constraint set is convex. We relax the constraints and define the Lagrangian [19], and we change $c_i \ge 0$ to $-c_i \le 0$

$$L(\mathbf{c}, \mu) = \sum_{i=1}^{N} \overline{\theta}_{i}(log(c_{i})) - \mu_{0} \left(\sum_{i=1}^{N} c_{i} - C\right) + \sum_{i=1}^{N} \mu_{i} c_{i}$$
(8)

with $\mu_0 \geqslant 0$ and $\mu_i \geqslant 0$, i = 1, ..., N. Next, we start with the stationarity condition of the Karush-Kuhn-Tucker (KKT) optimality conditions and we have

$$\nabla_{c_i} L(\mathbf{c}, \mu) = \frac{\overline{\theta}_i}{c_i} - \mu_0 + \mu_i = 0$$
 (9)

and since $\overline{\theta}_i>0$, we have that $\mu_0>\mu_i$, which means that $\mu_0>0$. Taking the complementary slackness conditions we have

$$\mu_0 \left(C - \sum_{i=1}^{N} c_i \right) = 0 \tag{10}$$

$$\mu_i c_i = 0 \tag{11}$$

$$\mu_0 \geqslant 0 \text{ and } \mu_i \geqslant 0, i = 1, ..., N$$
 (12)

As $\mu_0 > 0$, it follows from (10) that

$$\sum_{i=1}^{N} c_i = C \tag{13}$$

which means that c_i , i = 1, ..., N cannot be zero. Therefore by forcing $\mu_i = 0$, $\forall i = 1, ...N$ we have from (9)

$$c_i = \frac{\overline{\theta}_i}{\mu_0} \tag{14}$$

By combining (13) and (14) we reach the optimal solution which represents the weighted proportionally fair solution

$$c_i = \frac{\overline{\theta}_i}{\sum_{i=1}^N \overline{\theta}_i} C \tag{15}$$

Based on the proportionally fair bandwidth allocation of c_i to each UE_i, i=(1,...,N), we derive a ranking τ_{pf} of the UEs in the HetNet in descending order of allocated bandwidth. We

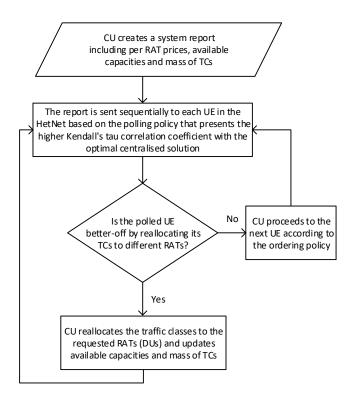


Fig. 1: Flowchart of polling functionality during a system cycle

denote as au_{pmp} the ranking that results from the dynamic PMP scheme we proposed, and we use Kendall's tau correlation coefficient $K(au_{pf}, au_{pmp})$ to compare the optimal centralized solution with our distributed scheme. Kendall tau distance reveals the pairwise disagreements between the two ranking lists.

D. Dynamic Selection of Distributed Policies

In our proposed framework in [4], the prices are updated periodically and whenever UEs enter or exit the area covered by the HetNet. We refer to the events that result in prices updates as cycles of the system. The practical need for sequential polling of the covered UEs to request for network resources led us to introduce three policies to define an appropriate priority in the ordering that the UEs are queried. In the first ordering policy τ_1 , the Network controller is ordering UEs in the HetNet based on their aggregate data rate demands of all traffic classes $\left(\sum_{l=1}^L r_i^l\right)$, in descending order. In the second policy τ_2 , the ordering is done in descending order of the normalized spectrum efficiency of the UEs, (w_i) . The third policy of ordering τ_3 , is based on the UEs' average sensitivity to the changes of the network conditions $\overline{\theta}_i$.

In this work, we provide a dynamic selection of ordering policy to achieve the closest ordering to the weighted proportionally fair approach, while maintaining the distributed nature of our framework. In every cycle, where prices are updated based on the congestion of the RATs, the Network controller calculates the correlation coefficients $K(\tau_{pf},\tau_j)$ for j=(1,2,3) and applies the policy with the maximum Kendall's tau correlation coefficient with the centralized

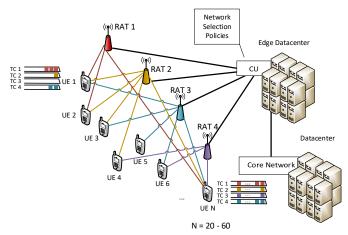


Fig. 2: Multi-RAT System Architecture

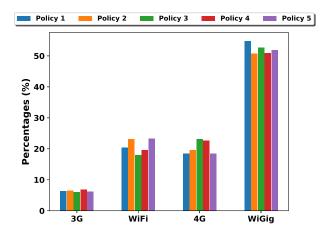
weighted proportionally fair approach. During a cycle the functionality of TC reallocation is summarized the flowchart provided in Figure 1, where every UE is polled and asked whether it is better-off by reallocating its TCs or not. When all UEs in the HetNet are polled, the system examines whether new UEs entered/left the HetNet, calculates the new policy rankings, and decides to change policy in case a different one presents higher Kendall's tau correlation coefficient.

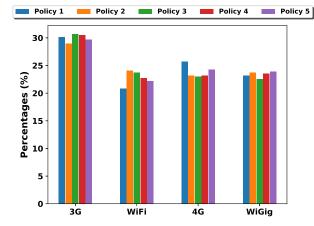
IV. EVALUATION

In this section, we provide a description of the system architecture and experimental evaluation of the proposed framework. First, we present the system components and the selected configurations and then we showcase results and insights of the experiments. Towards evaluating the capabilities of our proposed system in a large scale environment, we employed simulations rather than a real testbed environment as we did in our previous work [4] to avoid scale limitations. We developed a custom simulator in Python and we integrated it with the CU-DU implementation developed in [3]. Moreover, we organize our evaluation in five different experiments, during which the prices are constantly recalculated based on our proposed framework, and transmitted to the UEs of the network.

A. System Architecture

The overall system architecture that we investigate is depicted in Figure 2. We consider clients ranging from 20 to 60 UEs entering the Multi-RAT environment, whereas the base stations are disaggregated instances of DUs managed by a single CU. The CU is in charge of the selection of the policies for ordering the UEs and querying them for their TCs status and the preferred technology that will be used per each TC. We consider the existence of four different RATs: 1) the 3G RAT, with nominal capacity up to 42 Mbps, 2) the WiFi RAT, with capacity up to 130 Mbps for the IEEE 802.11n technology with 20MHz channel bandwidth and 2x2 MIMO configuration, 3) the 4G RAT with up to 70Mbps supported speeds, considering an LTE cell with 10MHz bandwidth and





- (a) Bandwidth demand distribution per each RAT
- (b) Utilization of each RAT for the exchanged traffic

Fig. 3: Experiment results for different policies regarding traffic distribution and utilization per each RAT

2x2 MIMO configuration, and 4) the WiGig RAT supporting up to 600 Mbps channel capacity. These measurements are derived by experimentally measuring the available technologies in the NITOS wireless testbed, an ecosystem that provides all these technologies for experimenting with [20].

TABLE I: Traffic Class configuration per each UE

TC Identifier	Throughput Range (Mbps)
TC1 - Background	0.3 - 1.0 Mbps
TC2 - Interactive	1.1 - 2.5 Mbps
TC3 - Streaming	2.6 - 8.0 Mbps
TC4 - Conversational	8.0 - 25.0 Mbps

Regarding the different traffic classes, we consider 4 different types with throughput values as indicated in Table I, based on [21] and each UE should be able to maintain up to 4 different TCs simultaneously. A representative scenario over which we evaluated our dynamic policy selection algorithm is the following: Each UE uniformly selects throughput values for each class from the ranges given in Table I. Initially, in our experiments we consider 20 UEs connected to the different available RATs in a random manner. During the execution of the experiment, we consider UEs entering the system and as they move, they may choose to update the allocation of technologies for their traffic classes according to the advertised prices, or leave the system. We consider 24 more UEs entering the system, 11 more passing through our system and 5 more being denied access by the network controller running at the CU side, as their needs cannot be covered by the system. This totals to an environment of up to 60 UEs actively participating in the evaluation of our scheme.

B. Experiments

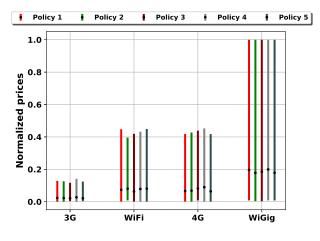
In this section, we present our experimental findings. We use five different policies for the UE ordering, with policies 1-3 being extensively discussed in [4]. Policy 1 orders the UEs based on the data rate demand of each client. Policy 2 applies ordering based on the spectrum efficiency of each client (e.g. how each client perceives the wireless channel as occupied by

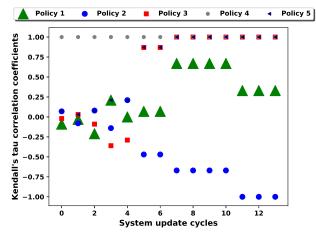
other users). Policy 3 is ordering the UEs of the network based on their perception of congestion in the network. Policy 4 is a solely centralized approach, which calculates the weighted proportionally fair bandwidth allocation of the connected UEs based on (15) and derive the ranking τ_{pf} , as indicated in Section III-C. Policy 5 is calculating Kendall's tau correlation coefficient [17] between policy 4 and policies 1-3, and selects the one with the optimal distance, i.e. higher correlation, every time that an event in the Multi-RAT system triggers a status update (periodically, when UEs enter/exit the HetNet).

Our goal is to examine how the different policies affect the distribution of the TCs to the available RATs of the system and how the overall throughput of the system is allocated to the available RATs depending on the applied polling policy. Figure 3 shows the distribution of overall exchanged bandwidth across the different RATs participating in our system and the utilization of each technology.

As we observe in Figure 3a, for all the policies that are applied, almost half of the total bandwidth is assigned to the WiGig technology. This is reflected also in Figure 3b, as we see that due to the wide capacity of the WiGig technology, it is utilized mainly for traversing traffic belonging to TC4. The selection of each RAT per each TC is taking into consideration the price allocated for using this technology. The price variations during our experiment are shown in Figure 4a. In order to compare the policies under examination, we normalize the costs based to the maximum price achieved on each experiment and the results of min, max and average values of the RAT prices, with the latter represented as dots.

Based on the above, we conclude that Policy 3 and Policy 5 (dynamic) are the ones that perform better in terms of price and fairness (as can be seen in Fig. 4). We clearly notice that this is also mirrored in the evaluation of the Kendall's tau correlation coefficient that is calculated by our algorithms in Figure 4b. After some initial iterations, Policy 3 is performing very close to the optimal (Policy 4), whereas Policy 2 is the worst in terms of fairness.





- (a) Normalized price variation per each RAT
- (b) Kendall's tau coefficient for the different policies

Fig. 4: Experiment results related to RAT pricing and Kendall's tau coefficient calculations

V. CONCLUSION

In this work, we developed an Ultra Dense Heterogeneous system that decides on how the UEs shall select the RATs for each of their traffic classes to be served based on different policies and dynamic pricing of each RAT. We modeled and evaluated different policies, based on the UE's data rate demands, spectrum efficiency and sensitivity to network conditions and compared them to the optimal, proportionally fair allocation. Our results showed that the dynamic application of these policies is able to achieve close to the optimal proportionally fair policy, as indicated by their Kendall's tau correlation coefficients. In the future, we foresee the experimental evaluation of these pricing schemes over a real testbed environment, and the application of novel mobility schemes for the clients entering and leaving the Multi-RAT system.

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